Cheap Talk Games with Two-Senders and Different Modes of Communication

ONLINE APPENDIX*

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A1 Introduction

The observed overtrust in our experiments shows that during the plays the receiver might have learned that the senders were somewhat overcommunicating. However, she does not seem to have fully exploited this knowledge, for the probability of trust is less than one, unlike implied by the best response correspondences calculated for the games in Section 2 of the main article. To explain this phenomenon that strategies with higher expected utilities are chosen with probabilities less than one, we will add noises to the payoff functions of the players, by following the logit-Agent Quantal Response Equilibrium (logit-AQRE) model of McKelvey and Palfrey (1998). This behavioral model assumes that each information set of a player is played by a different (hypothetical) agent. Each such agent will have responses in the form of

*We thank Santiago Sánchez-Pagés for sharing with us the software codes of the Benchmark Game in Sánchez-Pagés and Vorsatz (2007), which we have used in our experiments. We also acknowledge that this research was supported by TOBB University of Economics and Technology. The usual disclaimer applies.

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choice probabilities following a multi-nominal logit distribution, since the noise terms added to the payoff functions are independently and identically distributed according to the log Weibull distribution. Because of these noise terms, the responses of each agent will be smooth in the sense that each strategy which has a higher expected utility is played with a higher probability that is less than one.

The logit-AQRE model that we will formally define in Sections A2 and A3 for the Simultaneous and Sequential Games respectively allows the players to have not only different rationality levels but also different non-monetary costs of lying as in Peeters et al. (2013) and Gurdal et al. (2014). In Section A4, calculating the maximum likelihood estimations of the parameters of this model separately for the Simultaneous and Sequential Games, we will be able to investigate whether the observed behavior of players differing with respect to the mode of communication can be explained by their estimated rationality and cost parameters. Finally, we will conclude in Section A5.

A2 Simultaneous Mode of Communication

In order to apply the logit-AQRE model, we define two actions *truth* and *lie* for the two senders and two actions for the receiver in two situations: *trust to sender 1* and *distrust to sender 1* after observing *nonconflicting* and *conflicting* messages. For a sender, "truth" refers to sending the message that matches the actual payoff table and "lie" refers to doing the opposite. And, for a receiver, "trust to sender 1" refers to choosing the best response to the observed message of sender 1 and "distrust to sender 1" refers to choosing the opposite action. We would like to point out that if nonconflicting messages are observed and a receiver trusts to sender 1 this also implies that s/he trusts to sender 2. When conflicting messages are observed, trusting to sender 1 implies distrusting to sender 2 and vice versa. We denote the probability that sender 1 tells the truth by σ_1 and the same for sender 2 by σ_2. Similarly, we let σ_R^n stand for the probability of trusting to sender 1 upon observing nonconflicting messages and σ_R^c denote the probability of trusting to sender 1 upon observing conflicting messages. Then, sender 1 and sender 2 tell the truth with

$$\begin{aligned}
\sigma_1 &= \frac{e^{\gamma E[u_1(\text{truth})]}}{e^{\gamma E[u_1(\text{truth})]} + e^{\gamma E[u_1(\text{lie})]}} = \frac{1}{1 + e^{\gamma(E[u_1(\text{lie})] - E[u_1(\text{truth})])}}, \\
\sigma_2 &= \frac{e^{\gamma E[u_2(\text{truth})]}}{e^{\gamma E[u_2(\text{truth})]} + e^{\gamma E[u_2(\text{lie})]}} = \frac{1}{1 + e^{\gamma(E[u_2(\text{lie})] - E[u_2(\text{truth})])}}
\end{aligned}$$

Similarly, we have receiver trusting to sender 1 (after observing nonconflicting or conflicting messages) with probability

$$\sigma_R = \frac{e^{\gamma E[u_R(\text{trust to S1})]}}{e^{\gamma E[u_R(\text{trust to S1})]} + e^{\gamma E[u_R(\text{distrust to S1})]}} = \frac{1}{1 + e^{\gamma(E[u_R(\text{distrust to S1})] - E[u_R(\text{trust to S1})])}}$$

The parameter $\gamma \in [0, \infty)$ in the above expressions (as well as in the next section) can be positively associated with the rationality level of the players.[1] When γ is arbitrarily high, the players become fully rational and have standard best responses. On the other hand, when $\gamma = 0$, the players are fully irrational and act randomly.

Following Peeters et al. (2013) and Gurdal et al. (2014), we also assume that the senders have a non-monetary cost of lying denoted by c_i, $i = 1, 2$. Then,

Proposition A1. The unique logit-AQRE $(\sigma_1^*, \sigma_2^*, \sigma_R^{n*}, \sigma_R^{c*})$ of the simultaneous game solves the following four equations simultaneously:

$$\begin{aligned}
\sigma_1 &= \frac{1}{1 + e^{\gamma[4(\sigma_R^n + \sigma_R^c - 1) - c_1]}}, & \sigma_2 &= \frac{1}{1 + e^{\gamma[4(\sigma_R^n - \sigma_R^c) - c_2]}}, \\
\sigma_R^n &= \frac{1}{1 + e^{\gamma[8(1 - \sigma_1 - \sigma_2)]}}, & \sigma_R^c &= \frac{1}{1 + e^{\gamma[8(\sigma_2 - \sigma_1)]}}.
\end{aligned}$$

A3 Sequential Mode of Communication

Let σ_1 be the probability of sending truthful messages for sender 1, σ_2^t be the probability of sending truthful message for sender 2 after observing a truthful message of sender 1 and σ_2^l be the probability of sending truthful message for sender 2 after observing an untruthful message of sender 1. We let σ_R^n stand for the probability of trusting to sender 1 upon observing nonconflicting messages and σ_R^c represents the probability of trusting to sender 1 upon observing conflicting messages. We assume that senders' non-monetary cost of lying are denoted by c_1 for sender 1, c_2^t for sender 2 who has observed a truthful message by sender 1 and c_2^l for sender 2 who has observed untruthful message.

Proposition A2. The unique logit-AQRE $(\sigma_1^*, \sigma_2^{t*}, \sigma_2^{l*}, \sigma_R^{n*}, \sigma_R^{c*})$ of the sequential

[1] In fact, this parameter measures the precision of the probability density function associated with the noise term in each payoff function.

game solves the following five equations simultaneously:

$$\begin{aligned}
\sigma_1 &= \frac{1}{1+e^{\gamma[4(\sigma_R^n+\sigma_R^c-1)+4(\sigma_R^c-\sigma_R^n)(\sigma_2^l-\sigma_2^t)-c_1]}}, \\
\sigma_2^t &= \frac{1}{1+e^{\gamma[4(\sigma_R^n-\sigma_R^c)-c_2^t]}}, \quad \sigma_2^l = \frac{1}{1+e^{\gamma[4(\sigma_R^n-\sigma_R^c)-c_2^l]}}, \\
\sigma_R^n &= \frac{1}{1+e^{\gamma[8(1-\sigma_1-\sigma_2^l)+8\sigma_1(\sigma_2^l-\sigma_2^t)]}}, \quad \sigma_R^c = \frac{1}{1+e^{\gamma[8(\sigma_2^l-\sigma_1)+8\sigma_1(\sigma_2^t-\sigma_2^l)]}}.
\end{aligned}$$

A4 Maximum Likelihood Estimations

Now, we shall estimate the parameters of the logit-AQRE models we considered for the Simultaneous and the Sequential Game. We assume that the objective to be maximized in the Simultaneous Game is the log-likelihood function

$$L^{sim}(\lambda^{sim}, c^{sim}) = \sum_{s\in S^{sim}} n_s^{sim}\, ln(\sigma_s^{sim*}),$$

where S^{sim} = {truth-telling of sender 1, lie of sender 1, truth-telling of sender 2, lie of sender 2, receiver's trust when senders' messages are nonconflicting, receiver's distrust when senders' messages are nonconflicting, receiver's trust to sender 1 when senders' messages are conflicting, receiver's distrust to sender 1 when senders' messages are conflicting} denotes the collection of all strategies, n_s^{sim} denotes the number of times the strategy s has been chosen, and σ_s^{sim*} is the equilibrium probability of s in the Simultaneous Game given the rationality level λ^{sim} and the lying cost c^{sim} of the two senders.

The log-likelihood function to be maximized in the Sequential Game is

$$L^{seq}(\lambda^{seq}, c_1^{seq}, c_{2,t}^{seq}, c_{2,l}^{seq}) = \sum_{s\in S^{seq}} n_s^{seq}\, ln(\sigma_s^{seq*}),$$

where S^{seq} = {truth-telling of sender 1, lie of sender 1, truth-telling of sender 2 when sender 1 was truthful, lie of sender 2 when sender 1 was truthful, truth-telling of sender 2 when sender 1 lied, lie of sender 2 when sender 1 lied, receiver's trust when senders' messages are nonconflicting, receiver's distrust when senders' messages are nonconflicting, receiver's trust to sender 1 when senders' messages are conflicting, receiver's distrust to sender 1 when senders' messages are conflicting} denotes the collection of all strategies, n_s^{seq} denotes the number of times the strategy s has been chosen, and σ_s^{seq*} is the equilibrium probability of s in the Sequential Game given the rationality level λ^{seq}, the lying cost c_1^{seq} of sender 1, the lying cost $c_{2,t}^{seq}$ of sender 2 when sender 1 was truthful and the lying cost $c_{2,l}^{seq}$ of sender 2 when sender 1 lied.

Tables A1 and A2 present our estimation results for the rationality level, lying costs, and the expected utilities of the players in the Simultaneous and the Sequential Game, respectively. These two tables show that the hypothesis that 'the average bootstrapped value of the rationality parameter is zero' is rejected both in the Simultaneous Game (p-value: 0.05) and in the Sequential Game (p-value: 0.02), while λ^{sim} and λ^{seq} are not found to be statistically different (p-value: 0.71). Likewise, the hypothesis that 'the cost of lying is zero' is rejected for senders in the Simultaneous Game (p-value: 0.04) as well as for sender 2 in the Sequential Game when sender 1 lied (p-value: 0.03). In the Sequential Game, the same hypothesis cannot be rejected, however, for sender 1 (p-value: 0.28) or for sender 2 when sender 1 was truthful (p-value: 0.11).

Table A1. Logit-AQRE Estimation Results for the Simultaneous Game*

λ^{sim}	0.29 [0, 0.37] (0.19, 0.12)
c^{sim}	0.70 [0, 3.19] (1.87, 1.08)
Expected utility of each sender	2.41
Expected utility of receiver under nonconflicting messages	2.84
Expected utility of receiver under conflicting messages	2.48

* We exclude simultaneous plays in the Choice Treatment. In brackets, we report the 95 percent (standardized) confidence interval (obtained via bootstrapping with 1,000 repetitions using 70 percent of the experimental data). Below the brackets, we report the mean and the standard deviation of the bootstrapped parameters.

The results in Tables A1 and A2 also show that the expected utility of both sender 1 and sender 2 are higher in the Sequential Game than in the Simultaneous Game. Oppositely, the expected utility of the receiver is always lower in the Sequential Game. In addition, both in the Simultaneous and Sequential Game, the receiver becomes better off when the messages of the two senders are nonconflicting and becomes worse off otherwise. Below, we summarize these results.

Table A2. Logit-AQRE Estimation Results for the Sequential Game*

λ^{seq}	0.14 [0.05, 0.27] (0.14, 0.07)
c_1^{seq}	0.10 [0, 0.36] (0.08, 0.15)
c_{2t}^{seq}	0.85 [0, 1.43] (0.75, 0.61)
c_{2l}^{seq}	1.94 [0, 4.08] (2.65, 1.40)
Expected utility of sender 1	2.50
Expected utility of sender 2 when sender 1 was truthful	2.46
Expected utility of sender 2 when sender 1 lied	2.46
Expected utility of receiver under nonconflicting messages	2.61
Expected utility of receiver under conflicting messages	2.40

* We exclude sequential plays in the Choice Treatment. In brackets, we report the 95 percent (standardized) confidence interval (obtained via bootstrapping with 1,000 repetitions using 70 percent of the experimental data). Below the brackets, we report the mean and the standard deviation of the bootstrapped parameters.

Estimation Results. *Logit-AQRE estimations show that the subjects' rationality levels in the Simultaneous and Sequential Game are statistically the same and different from zero. Likewise, the cost of lying is statistically different from zero for senders in the Simultaneous Game and for sender 2 in the Sequential Game when* *sender 1 lied. In terms of expected utilities, the Sequential Game, as compared to the Simultaneous Game, makes both sender 1 and 2 better off while making the receiver worse off. In addition, in each game the receiver becomes better off when the two senders submit nonconflicting messages.*

A5 Conclusions

Maximum Likelihood Estimations (of the model in the main article) using the experimental data have showed that in the Simultaneous Game the presence of another sender does not eliminate a sender's intrinsic motive of truth-telling, recently observed in the laboratory experiments of Peeters et al. (2013) and Gurdal et al. (2014), both considering single sender-receiver games with simultaneous plays. However, in the Sequential Game sender 1 is found to be unburdened with the cost of lying, and this is also true for sender 2 when sender 1 was truthful. On the other hand, when sender 1 lied in the Sequential Game, sender 2 is observed to have a nonzero cost of lying. Evidently, for the Sequential Game one can argue that a sender will not have any intrinsic motives for truth-telling if and only if he knows that the other sender is likely (with some nonzero probability) to be truthful. Interestingly, we have also observed that the welfare of both senders are higher, while the welfare of the receiver is lower, in the Sequential Game than in the Simultaneous Game. This last finding suggests that the mode of communication may be a critical tool of design in principal-agent problems with multiple agents.

References

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[3] Peeters, R., Vorsatz, M., Walz, M., (2013). Truth, trust, and sanctions: On institutional selection in sender-receiver games, *Scandinavian Journal of Economics*, 115, 508-548.

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Appendix. Proofs of Propositions

Proof of Proposition A1. For sender 1 the expected payoff of choosing truth is as follows:

$$E[u_1(\text{truth})] = \sigma_2\left[\sigma_R^n 0.5 + (1-\sigma_R^n)4.5\right] + (1-\sigma_2)\left[\sigma_R^c 0.5 + (1-\sigma_R^c)4.5\right].$$

Similarly, expected payoff of choosing lie is:

$$E[u_1(\text{lie})] = \sigma_2\left[\sigma_R^c 4.5 + (1-\sigma_R^c)0.5\right] + (1-\sigma_2)\left[\sigma_R^n 4.5 + (1-\sigma_R^n)0.5\right] - c_1.$$

Then, $\sigma_1 = \frac{1}{1+e^{\gamma[4(\sigma_R^n+\sigma_R^c-1)-c_1]}}$.

Sender 2's expected payoff of choosing truth is:

$$E[u_2(\text{truth})] = \sigma_1\left[\sigma_R^n 0.5 + (1-\sigma_R^n)4.5\right] + (1-\sigma_1)\left[\sigma_R^c 4.5 + (1-\sigma_R^c)0.5\right].$$

Similarly, sender 2's expected payoff by choosing lie is:

$$E[u_2(\text{lie})] = \sigma_1\left[\sigma_R^c 0.5 + (1-\sigma_R^c)4.5\right] + (1-\sigma_1)\left[\sigma_R^n 4.5 + (1-\sigma_R^n)0.5\right] - c_2.$$

So, $\sigma_2 = \frac{1}{1+e^{\gamma[4(\sigma_R^n-\sigma_R^c)-c_2]}}$. Next, we find the expected payoff of a receiver who has observed the same -nonconflicting- messages sent by the two senders and trusts to sender 1.

$$E[u_R(\text{trust to S1})] = 9\sigma_1\sigma_2 + (1-\sigma_1)(1-\sigma_2).$$

Similarly, the expected payoff of a receiver who distrusts to sender 1 upon seeing nonconflicting messages is given by:

$$E[u_R(\text{distrust to S1})] = \sigma_1\sigma_2 + 9(1-\sigma_1)(1-\sigma_2).$$

We can conclude that $\sigma_R^n = \frac{1}{1+e^{\gamma[8(1-\sigma_1-\sigma_2)]}}$. The expected payoff of a receiver who trusts to sender 1 upon observing conflicting messages can be given as:

$$E[u_R(\text{trust to S1})] = 9\sigma_1(1-\sigma_2) + (1-\sigma_1)\sigma_2.$$

The expected payoff of a receiver who distrusts to sender 1 upon observing conflicting messages is:

$$E[u_R(\text{distrust to S1})] = \sigma_1(1-\sigma_2) + 9(1-\sigma_1)\sigma_2.$$

So, $\sigma_R^c = \frac{1}{1+e^{\gamma[8(\sigma_2-\sigma_1)]}}$. Finally, the uniqueness of the equilibrium is ensured by $\frac{\partial\sigma_1}{\partial\sigma_R^c} < 0$, $\frac{\partial\sigma_1}{\partial\sigma_R^n} < 0$ and $\frac{\partial\sigma_R^c}{\partial\sigma_1} > 0$, $\frac{\partial\sigma_R^n}{\partial\sigma_1} > 0$; $\frac{\partial\sigma_2}{\partial\sigma_R^n} < 0$, $\frac{\partial\sigma_2}{\partial\sigma_R^c} > 0$ and $\frac{\partial\sigma_R^n}{\partial\sigma_2} > 0$, $\frac{\partial\sigma_R^c}{\partial\sigma_2} < 0$. □

Proof of Proposition A2. Expected utility of sender 1 by being truthful is as follows:

$$E[u_1(\text{truth})] = \sigma_2^t \left[\sigma_R^n 0.5 + (1-\sigma_R^n)4.5\right] + (1-\sigma_2^t)\left[\sigma_R^c 0.5 + (1-\sigma_R^c)4.5\right].$$

Similarly, expected payoff of choosing to lie is:

$$E[u_1(\text{lie})] = \sigma_2^l \left[\sigma_R^c 4.5 + (1-\sigma_R^c)0.5\right] + (1-\sigma_2^l)\left[\sigma_R^n 4.5 + (1-\sigma_R^n)0.5\right] - c_1.$$

Thus, we get σ_1. Then, we derive the expected payoff of sender 2 by telling the truth after observing a truthful message of sender 1.

$$E[u_2(\text{truth})] = \sigma_R^n 0.5 + (1-\sigma_R^n)4.5.$$

Similarly, sender 2's expected payoff by choosing lie after observing a truthful message is:

$$E[u_2(\text{lie})] = \sigma_R^c 0.5 + (1-\sigma_R^c)4.5 - c_2^t.$$

So, we get that σ_2^t. Next, we find the expected payoff of sender 2 by telling the truth after observing an untruthful message of sender 1.

$$E[u_2(\text{truth})] = \sigma_R^c 4.5 + (1-\sigma_R^c)0.5.$$

Similarly, sender 2's expected payoff by choosing lie after observing an untruthful message is:

$$E[u_2(\text{lie})] = \sigma_R^n 4.5 + (1-\sigma_R^n)0.5 - c_2^l.$$

Thus, we arrive at σ_2^l. Now, we find the expected payoff of a receiver who trusts to sender 1 after observing nonconflicting messages:

$$E[u_R(\text{trust to S1})] = 9\sigma_1\sigma_2^t + (1-\sigma_1)(1-\sigma_2^l).$$

Similarly, the expected payoff of a receiver who distrusts to sender 1 upon seeing nonconflicting messages is given by:

$$E[u_R(\text{distrust to S1})] = \sigma_1\sigma_2^t + 9(1-\sigma_1)(1-\sigma_2^l).$$

And, we get that σ_R^n equals to the expression in the proposition. Finally, we calculate the expected payoff of a receiver who trusts to sender 1 upon observing conflicting messages.

$$E[u_R(\text{trust to S1})] = 9\sigma_1(1-\sigma_2^t) + (1-\sigma_1)\sigma_2^l.$$

The expected payoff of a receiver who distrusts to sender 1 upon observing conflicting messages is:

$$E[u_R(\text{distrust to S1})] = \sigma_1(1-\sigma_2^t) + 9(1-\sigma_1)\sigma_2^l.$$

Hence, we can find σ_R^c. Finally, the uniqueness of the equilibrium is ensured by $\frac{\partial\sigma_1}{\partial\sigma_R^c} < 0$, $\frac{\partial\sigma_1}{\partial\sigma_R^n} < 0$ and $\frac{\partial\sigma_R^c}{\partial\sigma_1} > 0$, $\frac{\partial\sigma_R^n}{\partial\sigma_1} > 0$; $\frac{\partial\sigma_2^t}{\partial\sigma_R^n} < 0$, $\frac{\partial\sigma_2^t}{\partial\sigma_R^c} > 0$ and $\frac{\partial\sigma_R^n}{\partial\sigma_2^t} > 0$, $\frac{\partial\sigma_R^c}{\partial\sigma_2^t} < 0$; and $\frac{\partial\sigma_2^l}{\partial\sigma_R^n} < 0$, $\frac{\partial\sigma_2^l}{\partial\sigma_R^c} > 0$ and $\frac{\partial\sigma_R^n}{\partial\sigma_2^l} > 0$, $\frac{\partial\sigma_R^c}{\partial\sigma_2^l} < 0$. $\square$